

Titres et résumés des conférences du colloque

Singularités et géométrie complexes

Lundi 24 Septembre

9h30-9h50 : A. Silva (Université de Rome). *Ouverture du colloque*

10h30-11h20 : B. Malgrange (Académie des Sciences / Institut J. Fourier). *Groupes différentiels algébriques*

11h30-12h20 : A. T. Huckleberry (Université de Bochum). *Cycles Galore*

14h30-15h20 : W. Veys (Université de Louvain). *Monodromy eigenvalues and zeta functions with differential forms*

Résumé. For a complex polynomial or analytic function f , there is a strong correspondence between poles of the so-called local zeta functions or complex powers $\int |f|^{2s} \omega$, where the ω are C^∞ differential forms with compact support, and eigenvalues of the local monodromy of f . In particular Barlet showed that each monodromy eigenvalue of f is of the form $\exp(2\pi\sqrt{-1}s_0)$, where s_0 is such a pole. We prove an analogous result for similar p -adic complex powers, called Igusa (local) zeta functions, but mainly for the related algebro-geometric topological and motivic zeta functions.

16h-16h50 : M. Toma (Institut Elie Cartan, Nancy). *Singular foliations on surfaces and invariant curves*

Résumé. We present a simple argument for the Theorem of Camacho-Sad, which shows the existence of invariant curves through any point of a holomorphically foliated non-singular complex surface. We further comment on extensions of this argument for two types of surface singularities.

Mardi 25 Septembre

9h-9h50 : J.-B. Bost (Université Paris XI, Orsay). *Géométrie arithmétique et géométries analytiques*

Résumé. In this talk, I will discuss several results in Diophantine geometry that assert the algebraicity of functions or subvarieties, *a priori* defined in formal geometry, under suitable hypotheses formulated in terms of analytic geometry and its avatars, such as rigid analytic geometry and Nevanlinna theory.

10h30-11h20 : A. Varchenko (Université de Caroline du Nord, Chapel Hill). *The B. and M. Shapiro conjecture in real algebraic geometry and the Bethe ansatz*

Résumé. I shall discuss the proof of Shapiro's conjecture by methods of math physics. Shapiro's conjecture says the following. If the Wronskian of a set of polynomials has real roots only, then the complex span of this set of polynomials has a basis consisting of polynomials with real coefficients. This statement, in particular, implies the following result.

If all ramification points of a parametrized rational curve $f : \mathbb{CP}^1 \rightarrow \mathbb{CP}^r$ lie on a circle in the Riemann sphere \mathbb{CP}^1 , then f maps this circle into a suitable real subspace $\mathbb{RP}^r \subset \mathbb{CP}^r$. The proof is based on the Bethe ansatz method in the Gaudin model. The key observation is that a symmetric linear operator on a Euclidean space has real spectrum.

11h30-12h20 : C. Hertling (Université de Mannheim). *Hermitian forms for isolated hypersurface singularities from tt^* geometry*

Résumé. The Brieskorn lattice of an isolated hypersurface singularity f gives by Fourier-Laplace transformation rise to a germ on $(\mathbb{C}, 0)$ of a vector bundle with a meromorphic connection with pole of order ≤ 2 at 0 with some real structure and some pairing. Such data together can be considered as a generalization of (mixed) Hodge structure. This point of view leads to tt^* geometry, which started with work of Cecotti+Vafa and Simpson.

It allows to generalize the variation of polarized mixed Hodge structures of Steenbrink and Varchenko on the μ -constant stratum to a variation of suitable data on the whole universal unfolding. Also Schmid's nilpotent orbits of Hodge structures generalize. For $r \gg 0$, the data of $r \cdot f$ give rise to a pure and polarized structure with a certain positive definite hermitian form. On a classifying space for Brieskorn lattices such forms induce a hermitian form with good curvature properties along horizontal directions.

14h30-15h20 : G. Tomassini (ENS Pise). *Cohomologie et extension*

16h-16h50 : J. Magnusson (Université de Reykjavik). *Un morphisme global de l'espace de Douady dans l'espace des cycles*

Résumé. On montre qu'il existe pour tout espace complexe un morphisme global de la réduction de son espace de Douady dans son espace des cycles. Ce morphisme est un prolongement du morphisme défini par D. Barlet (1976) sur le sous-ensemble de l'espace de Douady formé des sous-espaces de dimension pure. Dans le cas projectif ce morphisme coïncide avec le morphisme du schéma de Hilbert sur le schéma de Chow.

Mercredi 26 Septembre

9h-9h50 : C. Sabbah (Ecole Polytechnique, Palaiseau). *Développement asymptotique de distributions holonomes d'une variable complexe*

Résumé. Dans cet exposé, je donnerai la forme générale d'un germe de distribution holonome d'une variable complexe.

10h30-11h20 : M. Saito (RIMS, Kyoto). *Sur la b -fonction, le spectre et les idéaux multipliateurs d'une fonction*

Résumé. Quelques relations intéressantes entre la b -fonction, le spectre et les idéaux multipliateurs d'une fonction sont récemment découvertes. Cela nous permet par exemple de calculer la b -fonction d'un arrangement d'hyperplans dans quelques cas. On expliquera aussi le cas de sous-variétés de haute codimension s'il y a du temps.

11h30-12h20 : P. Schapira (Université de Jussieu, Paris). *Finiteness and duality on complex symplectic manifolds* (joint work with M. Kashiwara and J-P. Schneiders)

Résumé. On a complex symplectic manifold \mathfrak{X} , a W -algebroid stack $\mathcal{W}_{\mathfrak{X}}$ is an algebroid stack locally equivalent to the stack associated with a deformation quantization algebra. We prove that, under a natural properness condition, the composition $\mathcal{K}_1 \circ \mathcal{K}_2$ of two good kernels (*i.e.*, coherent and endowed with good filtrations) $\mathcal{K}_i \in D_{\text{gd}}^b(\mathcal{W}_{\mathfrak{X}_i \times \mathfrak{X}_{i+1}^a})$ ($i = 1, 2$) is a good kernel and that this composition commutes with duality.

As a particular case, we obtain that the triangulated category $D_{\text{gd,c}}^b(\mathcal{W}_{\mathfrak{X}})$ of good $\mathcal{W}_{\mathfrak{X}}$ -modules with compact supports is a Calabi-Yau triangulated category of dimension $d_{\mathfrak{X}}$.

We treat both the formal setting over the field $\widehat{\mathbf{k}} = \mathbb{C}((\hbar))$ and the analytic setting over \mathbf{k} , a subfield of $\widehat{\mathbf{k}}$, with two totally different proofs.

Jeudi 27 Septembre

9h-9h50 : A. Teleman (Université Aix-Marseille-I). *Moduli spaces of holomorphic bundles over class VII surfaces and applications*

Résumé. We introduce the concept of stability for holomorphic bundles over compact manifolds in the general (non-Kählerian) framework, and we explain how moduli spaces of poly-stable bundles over surfaces are identified with moduli spaces of instantons via the Kobayashi-Hitchin correspondence. We describe explicitly some moduli spaces of stable bundles over class VII surfaces and we show (using a combination of complex geometric and gauge-theoretical arguments) that any class VII surface with $b_2 = 2$ has a cycle of curves, hence it is a degeneration of a one-parameter family of blown up primary Hopf surfaces. This completes the classification problem for class VII surfaces in the case $b_2 = 2$, up to deformation equivalence.

10h30-11h20 : A. Parusinski (Université d'Angers). *On equivalence relations of real analytic function germs*

Résumé. We present a survey of new results on the blow-analytic classification of the real analytic function germs that include : a complete classification of two variable function germs, a construction of invariants that are based on the motivic integration on the space of real analytic arcs.

11h30-12h20 : F. Loeser (ENS Ulm, Paris). *On the Milnor fiber of composed polynomials*

Résumé. We shall present a series of results obtained in the last few years with Guibert and Merle on the Milnor fiber of composed polynomials. These results are motivic analogues of results by M. Saito, A. Némethi and J. Steenbink on the Hodge spectrum.

14h30-15h20 : H.-M. Maire (Université de Genève). *Monodromy and poles*

Résumé. Let $f : (\mathbb{C}^{n+p}, 0) \rightarrow (\mathbb{C}^p, 0)$ be a germ of holomorphic function. For any \mathcal{C}^∞ form φ of type (p, p) with compact support in \mathbb{C}^{n+p} , the fibre-integral $\int_{f=s} \varphi$ is of class \mathcal{C}^∞ outside the discriminant Δ of f . It has an asymptotic expansion near Δ when f defines an isolated complete intersection singularity. The case $p = 1$ is well understood.

We give effective information on this expansion when $p = 2$ in exhibiting obliques poles of the meromorphic current $\int |f|^{2\lambda}$.

16h-16h50 : B. Claudon (Institut Elie Cartan, Nancy). *Invariance des plurigenres du fibré canonique twisté*

Résumé. Soit $\pi : \mathcal{X} \rightarrow \mathbb{D}$ une famille de variétés projectives et (L, h) un fibré pseudo-effectif sur \mathcal{X} (i.e. la courbure $\Theta_h(L)$ de L est un courant positif fermé). L'objectif de cet exposé est de montrer comment adapter la méthode initiée par Y.-T. Siu puis largement simplifiée par M. Păun afin d'obtenir le résultat d'extension suivant : si l'idéal multiplicateur de $h_{\mathcal{X}_0}$ (restriction de h à la fibre centrale de la famille) est trivial, toute section de $m(K_{\mathcal{X}_0} + L)$ s'étend en une section de $m(K_{\mathcal{X}} + L)$; en d'autres termes, l'application de restriction

$$H^0(\mathcal{X}, m(K_{\mathcal{X}} + L)) \longrightarrow H^0(\mathcal{X}_0, m(K_{\mathcal{X}_0} + L))$$

est surjective.

Nous montrerons également que les techniques usitées permettent d'étendre des sections pluricanoniques L^2 dans les familles de revêtements universels.

Vendredi 28 Septembre

9h-9h50 : J. Włodarczyk (Université de Purdue). *Resolution algorithm in the characteristic zero and its possible extension in positive characteristic*

Résumé. We discuss the basic ideas of the Hironaka algorithm of the resolution of singularities in the characteristic zero and show how to extend them in positive characteristic.

10h30-11h20 : C. Soulé (IHES, Bures-sur-Yvette). *Sur les variétés sécantes de courbes projectives*

Résumé. Soit C une courbe lisse et projective, d un entier positif et Σ la variété d -sécante de C . On s'intéresse à borner la dimension des sous-espaces linéaires de Σ .

11h30-12h20 : Y.-T. Siu (Université de Harvard). *Multiplier ideal sheaves and their applications to analysis and algebraic geometry*

Résumé. Multiplier ideal sheaves identify the jet directions where estimates for partial differential equations fail. They were first introduced by Joseph J. Kohn to study the complex Neumann problem for weakly pseudoconvex domains and by Alan M. Nadel to study the existence of Kaehler-Einstein metrics for Fano manifolds. The technique of multiplier ideal sheaves injects in a new way methods of algebraic geometry into problems of analysis. It also opens new channels of applying analysis to problems in algebraic geometry, leading to the solution or partial solution of a number of longstanding open problems in algebraic geometry such as the Fujita conjecture, the deformational invariance of plurigenera, and the finite generation of the canonical ring.